AMD and JAM description

Akira Ono

Department of Physics, Tohoku University

Transport 2017: International Workshop on Transport Simulations for Heavy Ion Collisions under Controlled Conditions

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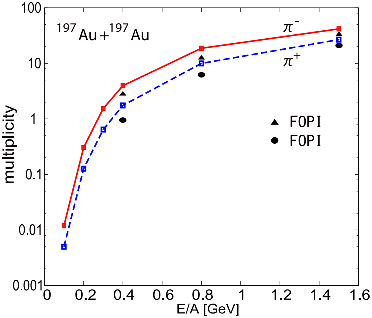
- JAM description
- AMD description

JAM Description

JAM: Jet AA Microscopic transport model

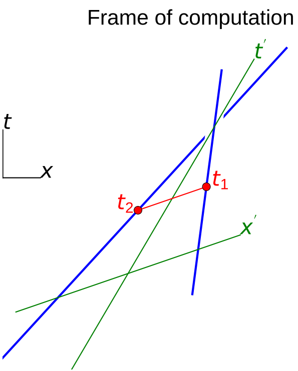
Y. Nara, N. Otuka, A. Ohnishi, K. Niita, S. Chiba, PRC61 (2000) 024901.

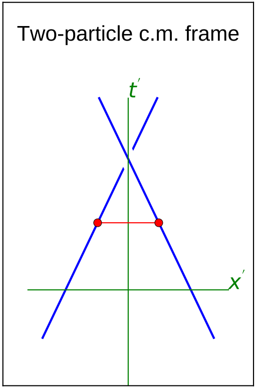
- Applied to high-energy collisions (1 ~ 158A GeV)
- Hadron-Hadron reactions are based on experimental data and the detailed balance.
- No mean field (default).
- Some improvements were introduced for the study of low-energy collisions (~ 300 MeV/u) by Ikeno, Ono, Nara, Ohnishi, PRC93 (2016) 044612.



Collision judgment in JAM

- There is no time step. ($\Delta t = 10$ or 20 fm/c for output)
- Each pair is checked for the minimum distance condition in each c.m. frame.
- A collision occurs at an equal time in the c.m. frame. Therefore $t_1 \neq t_2$.
- The order of collisions is determined by $\frac{1}{2}(t_1 + t_2)$.
- At every collision, the collisions in future are recalculated, of course.





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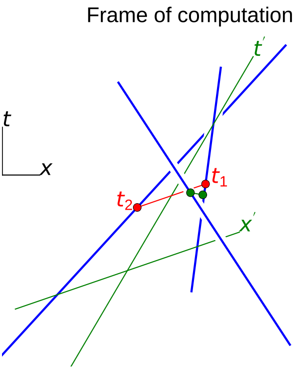
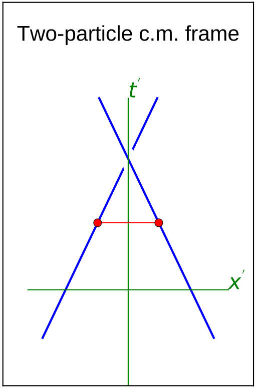




The k5 variable

Each particle carries a variable k5(i).

- When a collision/reaction/decay occurred, the final particles are given a common collision ID.
  k5(i) = 43576,   k5(j) = 43576
- Particles carrying the same k5 do not collide.
  if (k5(i).eq.k5(j)) cycle

Example:

- After $N+N \to N+\Delta$, this N and this Δ should not collide until one of them collide with some other particle.
- After $\Delta \to N+\pi$, this π should not be absorbed by this N until one of them collide with some other particle.

Box condition for the homework

Distance is redefined as suggested by the homework.

```
dr = r(i) - r(j)
dr = modulo(dr + L/2, L) - L/2
```

We did not moved the coordinates r(i) into the box.

~~r(i) = modulo(r(i), L)~~

We only did

```
write(*,*) modulo(r(i), L)
```

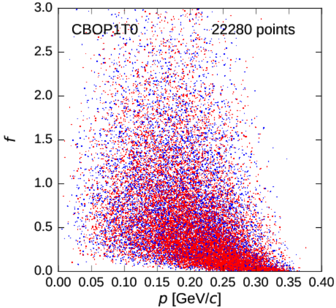
This is actually a good test to check whether all the distances are redefined, though we passed the test at the first try.

Pauli blocking based on the blocking factor

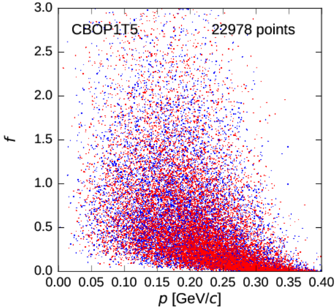
$$f_i = \frac{1}{2} \times 2^3 \sum_{j \in \tau_i (j \neq i)} e^{-(\mathbf{r}_i - \mathbf{r}_j)^2/4L - L(\mathbf{p}_i - \mathbf{p}_j)^2/\hbar^2}$$

$$\tau_i = \text{proton or neutron for particle } i$$

Homework 1 was very useful to check this Pauli blocking factor.



CBOP1T0 ($T = 0$ MeV)



CBOP1T5 ($T = 5$ MeV)

$NN \to N\Delta$ (Option Da)

$N + N \to N + \Delta(m)$

- m: mass of Δ ($m_N + m_\pi < m < \sqrt{s} - m_N$)
- $p_N(s)$ and $p_\Delta(m,s)$: initial and final momenta in the center-of-mass system

$$\frac{d\sigma[NN \to N\Delta(m)]}{dm} = \frac{C_I}{p_N s} \frac{|\mathscr{M}|^2}{16\pi} F(m) p_\Delta(m)$$

$$|\mathscr{M}|^2 = A \frac{s\Gamma_\Delta^2}{(s - m_\Delta^2)^2 + s\Gamma_\Delta^2}, \qquad F(m) = \frac{2}{\pi} \frac{m m_\Delta \Gamma(m)}{(m^2 - m_\Delta^2)^2 + m_\Delta^2 \Gamma(m)^2}$$

$$\Gamma(m) = \Gamma_\Delta \frac{m_\Delta}{m} \left(\frac{p_\pi(m)}{p_\pi(m_\Delta)} \right)^3 \frac{1.2}{1 + 0.2(p_\pi(m)/p_\pi(m_\Delta))^2}$$

$$m_\Delta = 1.232 \text{ GeV}, \quad \Gamma_\Delta = 0.118 \text{ GeV}, \quad \frac{A}{16\pi} = 64400 \text{ mb GeV}^2 \times R$$

$$C_I = \begin{cases} \frac{1}{4} & \text{for } nn \to n\Delta^0,\ np \to p\Delta^0,\ np \to n\Delta^+,\ pp \to p\Delta^+ \\ \frac{3}{4} & \text{for } nn \to p\Delta^-,\ pp \to n\Delta^{++} \end{cases}$$

We found that the sampling of m is not done accurately. (Phase III result will be updated.)

$$R = \int_{m_N+m_\pi}^{\sqrt{s}-m_N} F_3(m') p_\Delta(m') dm' \Big/ \int_{m_N+m_\pi}^{\sqrt{s}-m_N} F(m') p_\Delta(m') dm' \qquad \text{with} \quad F_3(m) = \frac{2}{\pi} \frac{m^2 \Gamma(m)}{(m^2 - m_\Delta^2)^2 + m^2 \Gamma(m)^2}$$

$N\Delta \to NN$ (Option Da)

Detailed balance, with the spin-isospin factor $g = 2(1+\delta_{NN})$

$$\sigma[N\Delta(m)\to NN] = \frac{1}{g} p_N^2 \times \mathrm{detbal} \times \int_{m_N+m_\pi}^{\sqrt{s}-m_N} \frac{d\sigma[NN\to N\Delta(m')]}{dm'} dm'$$

$$\mathrm{detbal} = \frac{1}{p_\Delta(m) \int_{m_N+m_\pi}^{\sqrt{s}-m_N} p_\Delta(m') F(m') dm'}$$

$$F(m) = \frac{2}{\pi} \frac{m m_\Delta \Gamma(m)}{(m^2 - m_\Delta^2)^2 + m_\Delta^2 \Gamma(m)^2}$$

The integrals of $p_\Delta(m')F(m')$ cancel so that we have

$$\sigma[N\Delta(m)\to NN] = \frac{C_I}{g} \frac{1}{p_\Delta(m) s} \frac{|\mathscr{M}|^2}{16\pi} p_N$$

$\Delta \leftrightarrow N\pi$ (Option Pa)

$\Delta(\sqrt{s}) \to N + \pi$

$$\Gamma(\sqrt{s}) = \Gamma_\Delta \frac{m_\Delta}{\sqrt{s}} \left(\frac{p_\pi(\sqrt{s})}{p_\pi(m_\Delta)}\right)^3 \frac{1.2}{1 + 0.2(p_\pi(\sqrt{s})/p_\pi(m_\Delta))^2},$$

$$m_\Delta = 1.232 \text{ GeV}, \quad \Gamma_\Delta = 0.118 \text{ GeV}$$

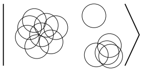
$$\Gamma[\Delta^-(\sqrt{s}) \to n + \pi^-] = \Gamma(\sqrt{s})$$
$$\Gamma[\Delta^0(\sqrt{s}) \to p + \pi^-] = \tfrac{1}{3}\Gamma(\sqrt{s})$$
$$\Gamma[\Delta^0(\sqrt{s}) \to n + \pi^0] = \tfrac{2}{3}\Gamma(\sqrt{s})$$
$$\Gamma[\Delta^+(\sqrt{s}) \to p + \pi^0] = \tfrac{2}{3}\Gamma(\sqrt{s})$$
$$\Gamma[\Delta^+(\sqrt{s}) \to n + \pi^+] = \tfrac{1}{3}\Gamma(\sqrt{s})$$
$$\Gamma[\Delta^{++}(\sqrt{s}) \to p + \pi^+] = \Gamma(\sqrt{s})$$

$N + \pi \to \Delta$

$$\sigma[N\pi \to \Delta(\sqrt{s})] = \frac{\pi}{p_\pi(\sqrt{s})^2} \times \frac{4}{2 \times 1} \times \frac{\Gamma(\sqrt{s})^2}{(\sqrt{s} - m_\Delta)^2 + \frac{1}{4}\Gamma(\sqrt{s})^2} \times \text{B.R.}$$

There is no free parameter once the choice of $\Gamma(\sqrt{s})$ is made.

Antisymmetrized Molecular Dynamics (very basic version)



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = (2.5 fm)$^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathscr{H}\}_{\text{PB}} \quad + \quad \text{(NN collisions)}$$

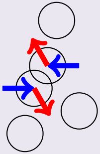
$\{\mathbf{Z}_i, \mathscr{H}\}_{\text{PB}}$: Motion in the mean field

$$\mathscr{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

H: Effective interaction (e.g. Skyrme force)

NN collisions

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

Antisymmetrized Molecular Dynamics (very basic version)

Wigner function for the AMD wave function

$$f_\alpha(\mathbf{r},\mathbf{p}) = 8\sum_{i\in\alpha}\sum_{j\in\alpha} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2} e^{-(\mathbf{p}-\mathbf{P}_{ij})^2/2\hbar^2\nu} B_{ij}B^{-1}_{ji}, \qquad \alpha = p\uparrow, p\downarrow, n\uparrow, n\downarrow$$

$$\mathbf{R}_{ij} = \frac{1}{2\sqrt{\nu}}(\mathbf{Z}_i^* + \mathbf{Z}_j), \quad \mathbf{P}_{ij} = i\hbar\sqrt{\nu}(\mathbf{Z}_i^* - \mathbf{Z}_j), \quad B_{ij} = e^{-\frac{1}{2}(\mathbf{Z}_i^* - \mathbf{Z}_j)^2}$$

Equation of motion for the wave packet centroids Z

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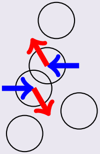


- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

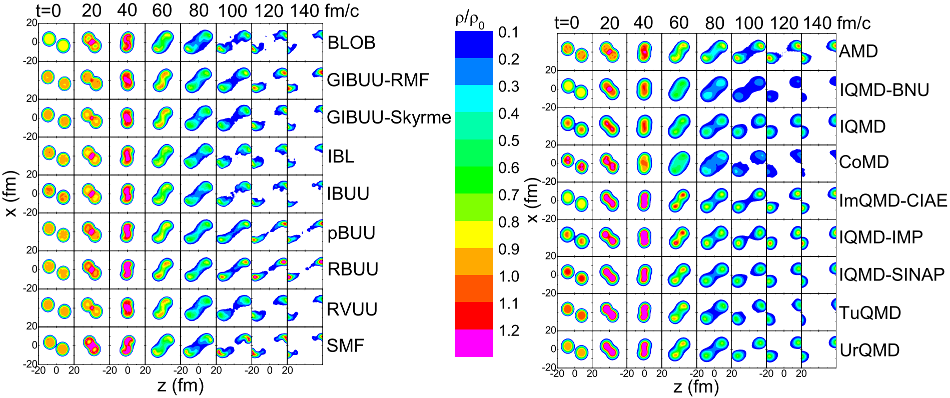
Transport code comparison

PHYSICAL REVIEW C **93**, 044609 (2016)

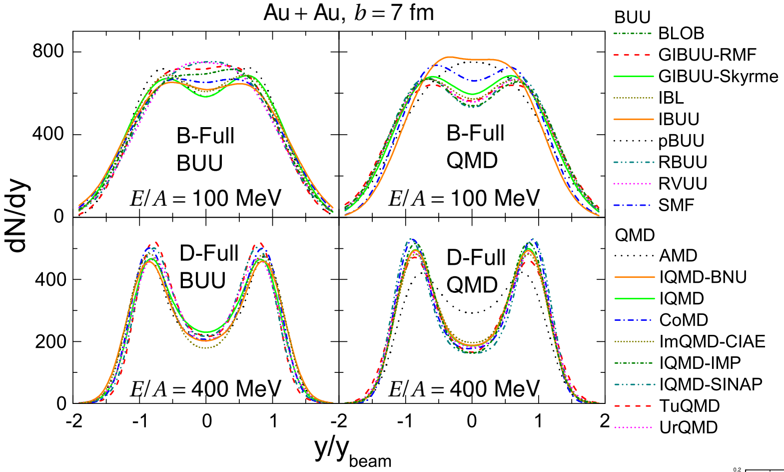
Understanding transport simulations of heavy-ion collisions at 100*A* and 400*A* MeV: Comparison of heavy-ion transport codes under controlled conditions

Jun Xu,[1,*] Lie-Wen Chen,[2,†] ManYee Betty Tsang,[3,‡] Hermann Wolter,[4,§] Ying-Xun Zhang,[5,‖] Joerg Aichelin,[6] Maria Colonna,[7] Dan Cozma,[8] Pawel Danielewicz,[3] Zhao-Qing Feng,[9] Arnaud Le Fèvre,[10] Theodoros Gaitanos,[11] Christoph Hartnack,[6] Kyungil Kim,[12] Youngman Kim,[12] Che-Ming Ko,[13] Bao-An Li,[14] Qing-Feng Li,[15] Zhu-Xia Li,[5] Paolo Napolitani,[16] Akira Ono,[17] Massimo Papa,[18] Taesoo Song,[19] Jun Su,[20] Jun-Long Tian,[21] Ning Wang,[22] Yong-Jia Wang,[15] Janus Weil,[19] Wen-Jie Xie,[23] Feng-Shou Zhang,[24] and Guo-Qiang Zhang[1]

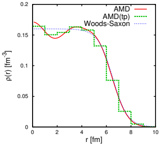
J. Xu et al., PRC93 (2016) 044609.



Transport code comparison: Rapidity distribution



- Weaker stopping in QMD than in BUU?
- Too strong stopping in AMD
  - ⇐ The proper density distribution is not used becuase the physical-coordinate approximation is not good.



Physical coordinate approximation in AMD

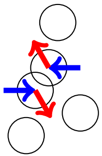
AMD doesn't usually use test particles.

For some purposes, the distribution function is approximated by a simple summation of Gaussians.

$$f(\mathbf{r},\mathbf{p}) = 8\sum_{i=1}^{A}\sum_{j=1}^{A} e^{-(\mathbf{r}-\mathbf{R}_{ij})^2/2\Delta x^2} e^{-(\mathbf{p}-\mathbf{P}_{ij})^2/2\Delta p^2} B_{ij}B^{-1}_{ji}$$

$$\approx 8\sum_{k=1}^{A} e^{-(\mathbf{r}-\mathbf{R}_k)^2/2\Delta x^2} e^{-(\mathbf{p}-\mathbf{P}_k)^2/2\Delta p^2} \qquad (\mathbf{R}_k,\mathbf{P}_k)\text{: physical coordinates}$$

- This physical-coordinate approximation is used in the traditional way of two-nucleon collisions in AMD.
  Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.
- However, this approximation is not necessarily good enough in some cases.



Test particles for the AMD wave function

Wigner function for the AMD wave function

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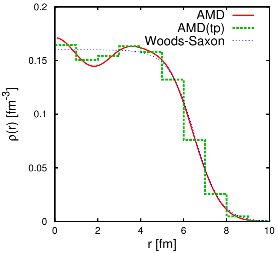
$$\mathbf{R}_{ij} = \frac{1}{2\sqrt{\nu}}(\mathbf{Z}_i^* + \mathbf{Z}_j), \quad \mathbf{P}_{ij} = i\hbar\sqrt{\nu}(\mathbf{Z}_i^* - \mathbf{Z}_j), \quad B_{ij} = e^{-\frac{1}{2}(\mathbf{Z}_i^* - \mathbf{Z}_j)^2}, \quad \Delta x^2 = \frac{1}{4\nu}, \quad \Delta p^2 = \hbar^2\nu$$

Regarding $f(\mathbf{r},\mathbf{p})$ as the probability distribution function, we randomly generate test particles $(\mathbf{r}_1,\mathbf{p}_1),(\mathbf{r}_2,\mathbf{p}_2),\ldots$
(1 test particle per nucleon)

- Density $\rho(r)$ for ^{197}Au (for test)
- Transport code comparison (for convenience)
- AMD + JAM (for pion production, Ikeno's talk)
- To improve NN collisions



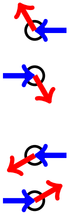
Improved NN collisions

More faithful to the BUU collision term.

$$I_{\text{coll}} = \int \frac{d\mathbf{p}_2}{(2\pi\hbar)^3} \int d\Omega\, |v| \left(\frac{d\sigma}{d\Omega}\right)_v \Big\{ f(\mathbf{r},\mathbf{p}_3) f(\mathbf{r},\mathbf{p}_4) [1 - f(\mathbf{r},\mathbf{p})][1 - f(\mathbf{r},\mathbf{p}_2)] - f(\mathbf{r},\mathbf{p}) f(\mathbf{r},\mathbf{p}_2) [1 - f(\mathbf{r},\mathbf{p}_3)][1 - f(\mathbf{r},\mathbf{p}_4)] \Big\}$$

1. Generate test particles $(\mathbf{r}_1,\mathbf{p}_1)$, $(\mathbf{r}_2,\mathbf{p}_2)$, ..., $(\mathbf{r}_A,\mathbf{p}_A)$
2. Collision attempt is judged by the Bertsch prescription, for each pair (i,j).

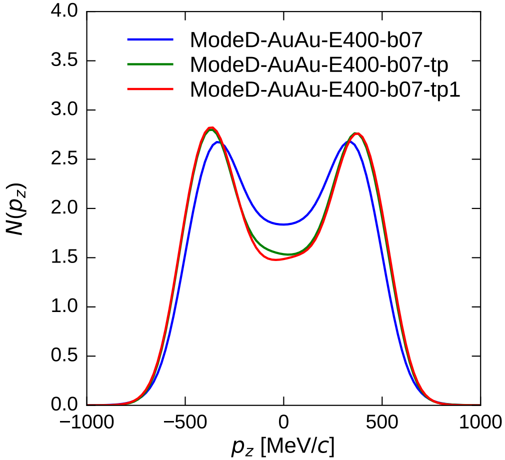
$$2|\mathbf{r}\cdot\mathbf{v}| < \mathbf{v}^2 \Delta t \quad \text{and} \quad \mathbf{r}^2 - \frac{(\mathbf{r}\cdot\mathbf{v})^2}{\mathbf{v}^2} < \frac{\sigma}{\pi}, \qquad (\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j, \quad \mathbf{v} = \mathbf{v}_i - \mathbf{v}_j)$$



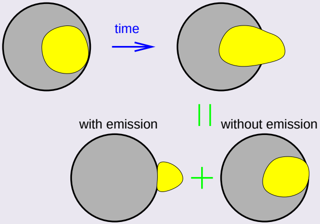
3. Pauli blocking/allowing by the factor

$$[1 - f(\mathbf{r}_i,\mathbf{p}'_i)][1 - f(\mathbf{r}_j,\mathbf{p}'_j)], \qquad \text{where} \quad \mathbf{p}'_i = \mathbf{p}_i + \mathbf{q}, \quad \mathbf{p}'_j = \mathbf{p}_j - \mathbf{q},$$

Effect in rapidity distribution



Two directions of extension of AMD

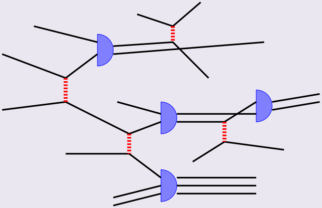


Wave-packet splitting: Give fluctuation to each wave packet centroid, based on the **single-particle motion**.

$$\frac{d}{dt}Z = \{Z, \mathcal{H}\}_{\mathrm{PB}} + \text{(NN Collision)} + \text{(W.P. Splitting)} + \text{(E. Conservation)}$$

Ono, Hudan, Chibihi, Frankland, PRC66 (2002) 014603

Ono and Horiuchi, PPNP53 (2004) 501



At each two-nucleon collision, **cluster formation** is considered for the final state.

$$\mathrm{N}_1 + \mathrm{B}_1 \;+\; \mathrm{N}_2 + \mathrm{B}_2 \;\to \mathrm{C}_1 + \mathrm{C}_2$$

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle \mathrm{CC}|V_{NN}|\mathrm{NBNB}\rangle|^2\delta(\mathcal{H} - E)$$

Ono, J. Phys. Conf. Ser. 420 (2013) 012103

Ikeno, Ono et al., PRC 93 (2016) 044612